OPTeSIM: Optimal Cellular Prepaid Plans for a Customer Base with eSIM-Capable Phones

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Abstract—The embedded subscriber identity module (eSIM) technology has the potential to revolutionize mobile subscribers' applications, transforming mobile network operators (MNOs) design and price their service offerings. As eSIM adoption accelerates, competition among MNOs, especially in prepaid markets, is intensifying. Traditional prepaid plans, based on fixed data and time allotments, often misalign with actual user consumption, leading to suboptimal performance. This paper presents OPTeSIM, a data-driven model for designing a finite set of optimal prepaid plans to maximize provider revenue. We model a user's consumption as an average usage rate (data/time) and posits that a user selects the plan with the closest matching plan usage rate. We formulate the design problem as the minimization of total customer loss, the monetized value of unused data or time, which is dependent on the probability density function (PDF) of customer usage rates. The necessary condition for optimality requires that each plan rate equalizes the probability mass of the user distribution it captures on its left and right boundaries. We propose a numerical algorithm that approximates this solution using historical usage data. The model's efficacy is demonstrated using two MNO datasets. For a real-world dataset of 0.5 million users, OPTeSIM plans captured 78% of the subscribers when offered alongside the MNO's 21 existing plans, resulting in a 3.6 fold increase in total profit. A second evaluation on a statistical dataset of one million users further confirmed that our model leads to improved profitability and higher user adoption compared to existing plans. The advantage grows with catalog size and approaches the performance of personalized pricing as the number of plans

Index Terms—eSIM, Prepaid Pricing, Data Driven Optimization, Churn.

I. Introduction

THE embedded subscriber identity module (eSIM) is a combination of the embedded universal integrated circuit card (eUICC) and cellular connectivity. It stores the mobile network operator (MNO) profiles, allowing users to access multiple MNOs simultaneously on a single device, and seamlessly switch between them. Between 2019 and 2022, the adoption rate of eSIM increased significantly, and projections indicate that around 98% of operators will offer eSIMs by 2025, and it is expected that approximately three billion smartphones will use this technology in the same year [1].

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The widespread adoption of eSIMs has led to increased competition within the telecommunications industry. The flexibility offered by eSIMs has disrupted traditional business models and increased competition between service providers [2], [3]. It has resulted in an increase in soft churn (switching between different plans from different providers), in addition to traditional churn (switching between providers) [4]. The disruptions caused by eSIMs can affect MNOs sales channels and customer relationships adversely. Moreover, in the saturated mobile market with new players such as mobile virtual network operators (MVNOs), MNOs should focus on customer retention as it is more challenging than acquiring new customers, especially when they are exposed to a wide array of options with increasingly competitive pricing.

Competition is further intensified in the prepaid market, as the eSIM adoption is expected to be more prevalent among prepaid users [5]. Therefore, to stand out from their competitors, service providers should develop innovative pricing strategies and value-enhancing offers to attract price-sensitive customers. Most of the existing prepaid pricing plans offered by MNOs around the world are based on tiered plans or their variants, such as bundled pricing. In this case, the data and time limit, as well as the price, are generally determined based on factors such as the target market, customer segmentation, competition, and the operator's own business goals. However, such plans exhibit suboptimal performance for both MNO and users, highlighting the necessity of designing service tiers that effectively balance the competing objectives of both parties.

In this paper, we investigate the issue of designing a finite number of optimal prepaid plans, with an aim of optimizing MNO's revenue. We first formulate a model to optimize the provider's utility while capturing the user's decision based on its usage rate. In the proposed model, OPTeSIM, we present a solution for the plan optimization problem based on the probability density function (PDF) of the customer's usage rate and numerical data. The evaluation results based on real-world datasets from two different MNOs demonstrate the benefits of the proposed approach over the traditional pricing schemes offered by the existing MNOs.

II. RELATED WORK

There has been extensive research on the pricing of mobile Internet services, leveraging network economic principles for resource management and profit optimization. Several static and dynamic pricing mechanisms have been proposed to optimize data service offerings, including recent research on addressing challenges related to pricing differential services [6]. However, despite theoretical interest, their practical application has been limited.

In contrast, limited research has been conducted on the composition and optimization of prepaid pricing plans, despite them being the predominant type of data pricing offered by MNOs at present. With respect to prepaid pricing, tiered pricing plans, defined based on different data and time limits, are the most prevalent type of plan. In addition, several MNOs offer bundled pricing plans, including offers on data, voice, text, and sponsored content from third-party providers [7]. Some research has focused on bundled data plans, data rollover [8], shared data plans [9], and specialized plans that cater to different user requirements [10]–[12]. In addition, [13] compared the usage-based and flat-rate pricing from the perspective of time-dependent pricing.

The authors in [14] introduced a dynamic programming model to consider users' consumption decisions, assessing which types of users benefit from data plans such as unlimited, prepaid, and usage-based plans. [15] proposed an economic model for tiered service networks using the Nash bargaining theory to determine optimal prices for service levels from both the user's and MNO's perspectives. Additionally, [16] proposed a personalized dynamic pricing scheme through the design of the product line to maximize operator total revenue by identifying optimal attribute levels for prepaid mobile Internet packages.

In eSIM research, the focus has primarily been on business aspects and churn prediction, with a specific emphasis on soft churn in [4]. This study introduces a personalized prepaid model for eSIM-based services, assessing its impact on customer churn behavior. Furthermore, the quality of service (QoS) aspects of personalized pricing have been highlighted in [2]. However, the existing literature lacks exploration of prepaid pricing in the context of finite offerings. In our study, our objective is to bridge this gap by designing optimal prepaid plans to maximize MNO's profit.

III. PROBLEM FORMULATION

We consider the pricing for a finite number of prepaid plans for a cellular provider. For each prepaid customer i, we compute the ratio of data used and time taken for each plan that the user has used so far. Then we average this value on all plans purchased by the user, which is denoted by x_i . We call this $average\ usage\ rate$ for the user i. For each plan j offered by a provider, we denote the ratio of data available on the plan and the duration of the plan by r_j . We call this $plan\ usage\ rate$. We assume that a user always chooses the closest matching plan. The closest offered plan, j^* , to user i's usage is given by

$$j^* = \underset{j}{\operatorname{argmin}} |r_j - x_i|. \tag{1}$$

If $x_i < r_{j^*}$, the user's plan will be time limited (running out of time before data). Meanwhile, if $x_i > r_{j^*}$, the user's plan will be data limited (running out of data before time). Note that $|r_{j^*} - x_i|$ represents the resources that were paid for by the user but were not used. We will refer to this as customer loss. The lower this value, the better for the customer and the more likely they will continue using the provider's plan. When multiple providers are available, the one that provides the plan with the lowest loss to the user will definitely be chosen by the user.

Suppose that a provider wants to develop n plans with the aim of minimizing the average customer loss (i.e., minimizing customer dissatisfaction). By reducing this metric, they will tend to avoid losing customers to competitors. Assume that we have sufficient data to determine the distribution of customer usage rates. Let \vec{r} denote the vector of plan rates and let f(x) denote the PDF of the customer usage rates. In order to handle edge cases, we use $r_0 = -r_1$ and $r_{n+1} = \infty$. We would like to find the plan rate vector that minimizes the total cost as

$$\vec{r}^* = \underset{\vec{r}}{\operatorname{argmin}} \sum_{j=0}^n \left\{ \int_{r_j}^{\frac{r_j + r_{j+1}}{2}} (x - r_j) f(x) dx + \int_{\frac{r_j + r_{j+1}}{2}}^{r_{j+1}} (r_{j+1} - x) f(x) dx \right\}.$$
(2)

Note that for case j=0, the first integral is from $-r_1$ to 0 which is zero since no samples have negative usage rates and the second integral goes from 0 to r_1 in which case all samples with usage rates between 0 and r_1 are mapped to r_1 as they should be (i.e., we cannot map to a zero usage rate plan). Similarly, when j=n the first integral goes from r_n to infinity since all such samples must map to r_n . The second integral has a zero range and hence is zero. Therefore, the solution to this problem will minimize the average customer loss.

IV. OPTIMAL PLAN DESIGN

In this section we provide a solution to the plan optimization problem. We assume that, given a sufficient number of samples, we can determine the PDF for customer usage rates. Suppose that we know the optimal values for r_{j-1} and r_{j+1} . Consider a variable r such that $r_{j-1} < r < r_{j+1}$ and L(r) represents the customer loss when $r_j = r$. Then, L(r) is given by

$$L(r) = \int_{r_{j-1}}^{\frac{r_{j-1}+r}{2}} (x - r_{j-1}) f(x) dx + \int_{\frac{r_{j-1}+r}{2}}^{r} (r - x) f(x) dx + \int_{r}^{\frac{r_{j-1}+r}{2}} (x - r) f(x) dx + \int_{r}^{\frac{r_{j-1}+r}{2}} (r_{j+1} - x) f(x) dx.$$
(3)

We can use the Leibniz integral rule to take the derivative of L(r) with respect to r, and we have

$$\frac{d}{dr}\left(\int_{a(r)}^{b(r)} g(r,x)dx\right) = g(r,b(r))\frac{d}{dr}b(r) - g(r,a(r))\frac{d}{dr}a(r) + \int_{a(r)}^{b(r)} \frac{\delta}{\delta r}g(r,x)d(x).$$
(4)

Using this rule, we can take the derivative of L(r) with respect to r as

$$L'(r) = \int_{\frac{r+r_{j-1}}{2}}^{r} f(x)dx - \int_{r}^{\frac{r+r_{j+1}}{2}} f(x)dx.$$
 (5)

Setting this derivative to zero i.e., L'(r) = 0, we find that a necessary condition for optimality is given by

$$\int_{\frac{r+r_{j-1}}{2}}^{r} f(x)dx = \int_{r}^{\frac{r+r_{j+1}}{2}} f(x)dx. \tag{6}$$

Taking the second derivative of L(r) with respect to r, we obtain

$$L''(r) = 2f(r) - \frac{1}{2} \left(f\left(\frac{r_{j-1} + r}{2}\right) + f\left(\frac{r_{j+1} + r}{2}\right) \right). \tag{7}$$

Note that for a large N,

$$\frac{1}{2}\left(f\left(\frac{r_{j-1}+r}{2}\right)+f\left(\frac{r_{j+1}+r}{2}\right)\right)\approx f(r)$$

and thus $L''(r) \approx f(r) > 0$. This optimality condition simply states that for a given plan usage rate, the density contained within the range of values captured on the left equals the density of the values captured on the right. Therefore, for all j, even for the cases j=0 and j=n, the optimal solution must satisfy

$$\int_{\frac{r_j + r_{j-1}}{2}}^{r_j} f(x) dx = \int_{r_j}^{\frac{r_j + r_{j+1}}{2}} f(x) dx.$$
 (8)

Given customer usage rates from a provider for their m prepaid customers, an approximate solution for the optimal plan rates can be obtained using Algorithm 1. The goal is to find the optimal plan rates based on the usage patterns of prepaid customers. We will use the number of samples within a range of usage rates as an approximation for the total density within the range.

Algorithm 1 begins by taking as input an ordered list of usage rates denoted as $\{x_1,\ldots,x_m\}$, where $x_i\leq x_{i+1}$ (see line 1). Initially, the plan rates r_j are set to zero for $j=1,\ldots,n$, where n represents the number of plans (see line 2). The algorithm then adjusts the plan rates r_j , using two variables v_l and v_r to track the number of customer samples captured to the left and right of the plan, respectively (see lines 3-5). The algorithm constructs each plan rate r_j based on the previous two rates r_{j-1} , r_{j-2} and the customer usage rates x_i . It iteratively adjusts the plan rates to balance the

distribution of samples captured on either side of the plan. By iteratively refining the plan rates based on the observed usage patterns, the algorithm aims to find a set of rates that effectively matches the distribution of usage across the customer base (see lines 6-17). Finally, the constructed set of plan rates $r_1, ..., r_n$ is returned (see line 18). Essentially, it ensures that the number of samples captured on the left of a plan is equal to the number captured on the right, effectively matching the distribution of usage across the customer base. This helps service providers in designing competitive pricing plans tailored to their customers' needs.

V. EVALUATION RESULTS

In order to compare the plans of two operators, we choose total profit as a performance metric. We follow the pricing model used by [4] as it was shown to provide good price estimates for actual plans. Suppose that a plan consisted of D MBs to be used over a period of T days. If the cost to the provider is α per MB and the cost to maintain the user on the network is β per day, the maximum cost of providing this plan would be $\alpha D + \beta T$.

We assume that the fixed one-time cost to sign up for the plan is negligible and hence it is ignored. Assume that the price charged is equal to the total cost plus some profit margin κ . The profit achieved for the plan is given by $\kappa(\alpha D + \beta T)$. We can divide by T to obtain a daily profit of $\kappa(\alpha r + \beta)$ where r = D/T is the usage rate of the plan.

Algorithm 1 Determination of optimal plans

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1: \{x_1, \ldots, x_m\} where x_i \leq x_{i+1} (ordered usage rates)
 2: Set r_i = 0, \quad j = 1, \dots, n (initialize plan rates)
 3: Set s = 0
 4: Set v_l = 0 (samples captured to left of plan n)
 5: Set v_r = m (samples captured to right of plan n)
 6: while (v_r > v_l) do
          s = s + 1
 7:
          r_1 = x_s
          r_2 = 2x_{2s} - r_1
          for i = 3 : n do
10:
               Let k be such that r_{j-1} \in [x_k, x_{k+1}]
Let c = number of samples in \left(\frac{r_{j-2}+r_{j-1}}{2}, r_{j-1}\right]
11:
12:
                r_i = 2x_{c+k} - r_{i-1}
13:
14:
          \begin{array}{l} v_l = \text{number of samples in interval } \left(\frac{r_{n-1} + r_n}{2}, r_n\right] \\ v_r = \text{number of samples in interval } (r_n, \infty] \end{array}
15:
16:
17: end while
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This, however, assumes that the user consumed all assigned data over the entire allocated period. This is not typically the case. Suppose that the user ran out of time and so used d < D data; then they would have ended up paying for resources that

18: return \vec{r}

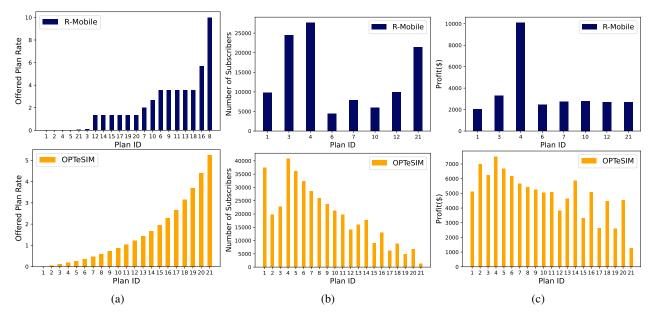


Fig. 1: (a) Offered plans, (b) number of subscribers, and (c) profit for R-Mobile and OPTeSIM.

were not used, contributing to additional "revenue" for the provider (customer loss). If x=d/T, this excess per day is given by

$$(\alpha r + \beta) - (\alpha x + \beta) = \alpha (r - x).$$

Similarly, if t < T, the excess revenue is given by

$$(\alpha r + \beta) - (\alpha r + \beta t/T) = \frac{\beta}{r}(x - r).$$

Therefore, we can write the overall monthly profit as

$$P = \kappa(\alpha r + \beta) + \max\left\{\alpha(r - x), \frac{\beta}{x}(x - r)\right\}. \tag{9}$$

Let S_1 and S_2 denote the set of users assigned to the plans of provider 1 and provider 2, respectively, and $\mu = \alpha/\beta$. The ratio of profits of these two providers is given by

$$\frac{P_1}{P_2} = \frac{\sum_{i \in S_1} \kappa(\mu r_i + 1) + \max\left\{\mu(r_i - x_i), \frac{1}{x_i}(x_i - r_i)\right\}}{\sum_{i \in S_2} \kappa(\mu r_i + 1) + \max\left\{\mu(r_i - x_i), \frac{1}{x_i}(x_i - r_i)\right\}}.$$
(10)

A. Evaluation Based on Real-World Dataset

The evaluation is based on user usage rate data (collected over a period of six months) for about 0.5 million users from a real MNO offering 21 different prepaid plans. Since these data and their sources are confidential, we will refer to this MNO as "R-Mobile" in the subsequent discussion.

In Figure 1(a), we plot the rates offered for 21 plans offered by R-Mobile and OPTeSIM. It can be observed that OPTeSIM offers a wider range of data rates compared to R-Mobile, due to the fact that the user's usage rate is considered when designing the offered plan rate in OPTeSIM. Supposing that both MNO plans and the optimal plans were offered, we can determine the users that choose each of the plans using

equation (1). With 42 available plans (21 each from R-Mobile and OPTeSIM), we assume that each user will choose the plan that offers a rate closest to their usage rate. The number of subscribers per plan is illustrated in Figure 1(b). Out of a total of 528,232 users, about 78% of the users subscribed to the OPTeSIM plans. Furthermore, unlike OPTeSIM, where all 21 plans were chosen, only 8 of 21 plans offered by R-Mobile were chosen by users, and the remaining 13 plans remained unused. Next, we compared the profit achieved using (9), with $\alpha = 0.8$, $\beta = 0.9$, estimated applying linear regression on actual plans. The value of κ is assumed to be 0.125 based on the average profit margin for the telecommunication industry [17]. From Figure 1(c), it can be found that although OPTeSIM offers lower plan rates, it attracted more users as the plans were designed optimally, and thus the overall profit of OPTeSIM was about 3.6 times higher than that of R-Mobile.

B. Evaluation based on Statistical Data

To further validate the robustness of OPTeSIM, we also performed evaluations based on statistical information from another MNO provided in [18]. The dataset used includes one million users from China Telecom (which we refer to as C-Mobile) and six prepaid plans are offered. Since raw data are not available, we used the statistical data provided in [18] to generate user usage rates. We used an exponential distribution along with the provided statistics to model the usage rate distribution.

We generated one million users and determined the six optimal plans using Algorithm 1, as illustrated in Figure 2(a). Even for a small number of plans, the plan rates designed using OPTeSIM can be optimized based on the user's usage rates. For instance, let us compare the smallest plans offered by C-Mobile and OPTeSIM. The C-Mobile plan provides 500MB of data, whereas the OPTeSIM plan offers 200MB.

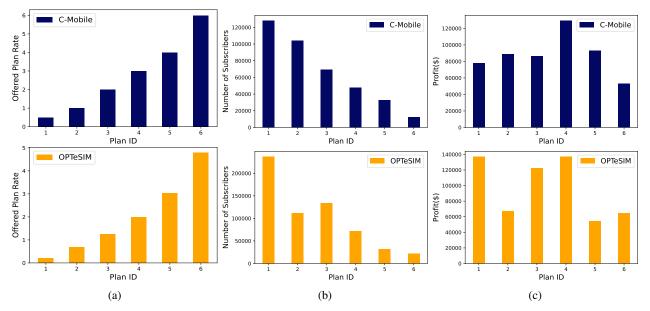


Fig. 2: (a) Offered plans, (b) number of subscribers, and (c) profit for C-Mobile and OPTeSIM.

Meanwhile, the OPTeSIM plan attracted 236,592 subscribers, nearly double the 127,991 users who opted for the C-Mobile plan. This significant difference in the number of subscribers indicates that by optimally designing the plans, OPTeSIM successfully attracted a larger segment of users with lower data usage rates. In Figure 2(b) we plot the number of users who subscribe to each plan for C-Mobile and OPTeSIM. It can be seen that out of one million users, 60.5% of the users subscribed to the OPTeSIM plans, while only 39.5% users subscribed to the C-Mobile plans. This is attributed to the fact that the offered OPTeSIM plan rates match their usage rates more closely compared to the C-Mobile plans. We also plotted the profit achieved per plan in Figure 2(c). It can be seen that the total profit for OPTeSIM is 1.03 times the profit for C-Mobile. Although the benefit of optimization is limited for such a small number of plans, the profit ratio increases with increasing the number of plans, as illustrated in the next subsection.

C. Effect of Number of Plans on Profit

In the near future, as the competition intensifies further due to the widespread adoption of eSIMs, and the users demand greater customization and flexibility, even prepaid plans will increasingly need customized designs to cater to the personalized requirements of users. To highlight this capability in OPTeSIM, in Figure 3, we plot the profit ratio for OPTeSIM in R-Mobile and C-Mobile, as the number of OPTeSIM plans offered increases (until all customers are acquired by OPTeSIM). As the number of plans increases, OPTeSIM will naturally capture all customers and therefore the profit advantage will increase. However, note that the revenue due to the customer loss goes to zero since each customer eventually gets exactly what they need. This limit is what was termed

optimal personalized pricing in [4]. We find that in the limit, OPTeSIM plans approach the personalized pricing limit.

VI. CONCLUSION

As eSIM-capable phones proliferate, users can effortlessly switch between providers with prepaid subscriptions, leading to soft-churn concerns. Hence, optimal pricing is crucial for operators to prevent customer loss due to high costs or financial loss from underpricing. We explore pricing optimization for increased profitability for finite plan offerings, showcasing the approach with real data. Future work aims to expand this optimization to plans that incorporate not only data and time limits but also QoS constraints such as latency and throughput.

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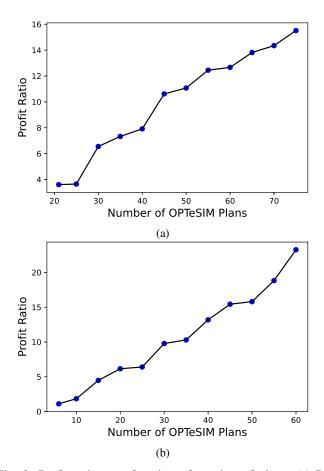


Fig. 3: Profit ratio as a function of number of plans: (a) R-Mobile and (b) C-Mobile.

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