Optical Lens Inventory Optimization Model

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***Abstract*—We provide an optimization model for optical lens inventory management. This model aims to reduce costs by predicting which lenses should be pre-ordered in bulk while ordering less popular lenses through a third party which is more costly. The model takes into account prediction errors. If too many lenses of a certain type are pre-ordered then there is an associated cost because funds were invested. If too few lenses of a certain type are pre-ordered then for some customers the lens will have to be ordered through a third party which will be an added cost to the chain. Taking into account these prediction errors we use historical data to determine what should be ordered. We compare the proposed approach with the approach that was used by the chain and demonstrate significant cost savings.**

Keywords: Forecasting, Prophet, ARIMA, Linear Programming, Inventory Management, Optimization, Optical Chain

I. Introduction

Small optical chains face a critical challenge, "balancing the cost benefits of stocking common prescription lenses against the capital tied up in inventory. Current industry practices often involve either complete reliance on expensive third-party optical labs (middlemen), which can impose markups as high as 100-300%, or intuition-driven procurement. Both approaches frequently lead to suboptimal outcomes, including lost profit, stockouts, or excessive inventory". [1]

However, many small optical chains (including the one in this study) employ dispensing opticians who are capable of fitting lenses into frames themselves, eliminating the need for external lab services for fitting and surfacing, and incurring no additional cost to the company for this step. By stocking lenses, these companies can offer same-day or next-day service for common prescriptions and coatings. This not only enhances patient satisfaction but also provides a substantial competitive advantage by reducing the typical two-week wait.

We present a data-driven framework to overcome these challenges. It addresses the key question: *given a quarterly budget, B, what quantity of each lens combination should be purchased to maximize profit, accounting for a 10% profit margin on direct sales, middleman fees for unfulfilled demand, and holding costs for inventory?* Our framework forecasts demand for the most frequently ordered lens combinations using two advanced time-series models (Prophet and ARIMA), optimizes procurement via linear programming to maximize profit under various budget constraints, and incorporates real-world prescription distribution patterns to determine the quantities of each lens combination and prescription to be ordered.

II. Literature Review

Accurate demand forecasting is critical across various sectors including manufacturing, healthcare and energy to ensure optimized resource allocation, strategic planning and efficient operations.

This literature review covers recent advancements in time series forecasting methodologies, with a focus on the application and comparative performance of Autoregressive Integrated Moving Average (ARIMA) and Prophet models [1].

Research has consistently utilized ARIMA models as a robust benchmark in time series forecasting. For instance, Sharma and Mishra [2] successfully employed the ARIMA forecasting model to predict future electricity demand for both domestic and commercial purposes, demonstrating its capability in generating forecasts with low error values. Similarly, Pindiga et al. [3] applied ARIMA models in the financial sector for predicting stock indices, showcasing their continued relevance across various forecasting applications.

The Prophet model, recognized for its scalability and ability to manage complex seasonal patterns, trends, and piecewise trends, has also found significant application across different fields. According to Chitwadgi [4], its features, such as automatic trend detection and handling of multiple seasonalities, make it a valuable tool for complex time series data. Chitwadgi [4] also explored Prophet for production planning within manufacturing environments, a context where this approach was noted as previously limited, yet proved itself capable with seasonal and trend characteristics of production datasets. Furthermore, Pindiga et al. [3] also used Prophet for time-series forecasting to predict stock indices, similar to ARIMA, indicating its versatility in financial markets.

Several comparative studies have shed light on the relative performance and specific strengths of ARIMA and Prophet. Chitwadgi [4], explored the application of the Prophet forecasting model to production planning in a manufacturing environment, with its performance compared against both ARIMA and XGBoost models. This research aimed to determine the most accurate technique for production needs and their findings revealed that while XGBoost outperformed ARIMA, Prophet achieved greater accuracy in capturing the seasonal patterns and complexities of their manufacturing dataset.

Menculini et al. [5] conducted a direct comparison of Prophet, ARIMA, and deep learning models for forecasting wholesale food prices. Their findings indicated that while ARIMA and LSTM neural networks performed similarly, Prophet offered a quicker and easier implementation. However, as observed by Menculini et al. [5], Prophet was considerably less accurate in this specific context of wholesale food price prediction, suggesting that its suitability can be highly dependent on the particular data characteristics.

Duarte et al. [6] compared time-series prediction models, including ARIMA and Prophet, for healthcare emergency department indicators, particularly in light of the impact of the COVID-19 pandemic. Such comparisons, according to Duarte et al., are crucial for providing reliable predictions to support acute unit planning and resource optimization in dynamic healthcare environments.

As proposed by Stefenon et al. [7], a hybrid approach combining seasonal and trend decomposition (utilizing LOESS) with the Prophet methodology was used for more accurate and resilient time series forecasting of Italian electricity spot prices. This model demonstrated improved forecast accuracy and a reduction in the mean absolute percentage error (MAPE) by 18% compared to previous methods, showcasing the potential benefits of combining Prophet with other techniques.

These comparative analyses highlight that while ARIMA remains a reliable statistical model capable of capturing linear patterns, Prophet often provides a more automated and robust approach for handling complex seasonalities and trends, along with greater ease of use, though its accuracy can vary depending on the specific application and data characteristics. The emergence of hybrid models also suggests that combining the strengths of different forecasting techniques can lead to superior predictive performance in complex scenarios [8].

III. Methodology

This study addresses the core inventory optimization challenge: *Given a fixed quarterly budget, how should lens inventory be procured to maximize profit while minimizing middleman reliance?* Figure 1 outlines our four-phase approach:

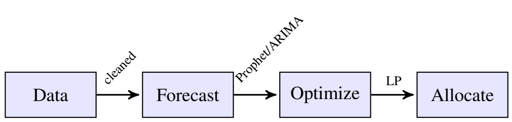

Fig. 1: Methodology flow

A. Dataset Description and Preparation

The dataset comprised 22 quarters (approximately 5.5 years) of historical sales data from a single branch of a local optical chain. Key features included transaction dates (specifically 'Date of write up'), Branch, and 'Job Description', which detailed the lens combination sold.

A second, manually compiled dataset (cost_data_df) provided essential financial information for various lens combinations. This included: Uncut_lens_cost (the direct cost of purchasing an uncut lens from a manufacturer), Middleman_fee (the cost incurred when a lens is procured through an optical lab middleman), Holding_cost_per_unit (the estimated cost of storing one unit of inventory for one quarter.)

The Uncut_lens_cost was obtained from a direct supplier [9] and the Middleman_fee from a local optometric practice.

The raw sales data, originally recorded by hand, was subject to initial data entry into Google Sheets. The data cleaning process involved: Standardizing lens combination names (e.g., 'sv, trans' vs 'sv, transition'), imputing or removing entries with incomplete transaction information where appropriate.Quarterly aggregation of sales data to align with the proposed procurement cycle, which also helped mitigate the impact of sparse daily or weekly data points for less common items.

The initial dataset contained a large number of unique lens combinations. To focus on the most impactful items for inventory optimization, we identified the top 30 lens combinations by total sales volume. These top 30 combinations accounted for approximately 72.50% of the overall sales data, providing a practical balance between coverage and computational manageability given the limited historical data for very low-volume items.

B. Forecasting

Two distinct time-series forecasting models were employed: Prophet and ARIMA.

1) Prophet Model [10], [11]: The Prophet model, developed by Meta (Facebook), is designed for forecasting time series data with strong seasonal effects and trends. It models time series as an additive regression model:

$$y(t) = g(t) + s(t) + h(t) + \epsilon_t$$

where:

- $g(t)$ is the trend function (piecewise linear or logistic growth).
- $s(t)$ represents periodic changes (e.g., weekly, yearly seasonality).
- $h(t)$ represents the effects of holidays.
- ϵ_t is the error term.

For this implementation, Prophet was configured with:

- **changepoint_prior_scale = 0.01**: A smaller value to make the trend more stable and less prone to overfitting abrupt changes.
- **seasonality_prior_scale = 10**: A larger value to allow for more flexibility in modeling the seasonal components, reflecting the expected strong seasonality in optical sales.
- **Logistic growth with capacity scaling**: Applied for some combinations where demand might plateau due to market saturation or physical constraints.

2) ARIMA Model [12], [13]: For computational efficiency, a simplified grid search was performed over the (p,d,q) parameters for each series, restricting p, d, and q to values in $\{0,1\}$. This approach aimed to identify reasonable ARIMA configurations without exhaustive computation, which would be prohibitive for 30 distinct series.

C. Performance Evaluation

Rolling origin evaluation was utilized for both models. This method involves training the model on an initial window of data, forecasting for the next period, and then "rolling" the origin forward by including the actual data from the forecasted period in the training set for the next iteration. This simulates a real-world forecasting scenario and provides robust estimates of model performance. Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) were used as the primary evaluation metrics (as presented in Table 1 in the Results section).

D. Optimization Model

The core of the inventory optimization is a **linear programming (LP)** model, implemented using the PuLP solver in Python. The objective is to maximize the total profit over the forecasting horizon, taking into account direct sales profit, middleman costs, and holding costs. Crucially, the model assumes **no profit is made on lenses sourced from the middleman**. This is a key business assumption embedded in the objective function, heavily incentivizing the model to utilize its budget for direct procurement to fulfill demand. Using the middleman becomes a "last resort" to satisfy customer demand when direct procurement is insufficient, actively reducing total profit.Let:

- t be the time period (quarter).
- k be the lens combination index.
- $D_{t,k}$ be the forecasted demand for lens combination k in quarter t.
- C_k^c be the direct unit cost of uncut lens combination k.
- C_k^m be the middleman unit cost for lens combination k.
- C_k^h be the holding cost per unit for lens combination k per quarter.
- B_t be the budget for quarter t.
- $s_{t,k}$ be the inventory level of lens combination k at the end of quarter t.
- $q_{t,k}$ be the quantity of lens combination k ordered directly in quarter t.
- $f_{t,k}$ be the quantity of lens combination k fulfilled from direct stock in quarter t.
- $\pi = 0.10$ be the profit margin on direct sales.

The function to be maximized is given by::

$$\text{Maximize} \sum_{t,k} \left[\pi C_k^c f_{t,k} - C_k^m \max(0, D_{t,k} - f_{t,k}) - C_k^h s_{t,k} \right] \tag{1}$$

where $\max(0, D_{t,k} - f_{t,k})$ represents the units sourced from the middleman.

The constraints are given by:

1) **Inventory Balance:** The inventory at the end of a quarter is the previous quarter's inventory plus new orders, minus fulfilled demand.

$$s_{t,k} = s_{t-1,k} + q_{t,k} - f_{t,k}$$

(Initial inventory $s_{0,k}$ is assumed to be zero for simplicity, or can be set to known starting levels)

2) **Budget Constraint:** The total cost of directly ordered lenses in any given quarter cannot exceed the allocated quarterly budget.

$$\sum_k (q_{t,k} \cdot C_k^c) \leq B_t$$

3) **Fulfillment Bounds:** Demand can only be fulfilled up to the available stock (previous inventory + new orders) and cannot exceed the total demand.

$$f_{t,k} \leq D_{t,k}$$

$$f_{t,k} \leq s_{t-1,k} + q_{t,k}$$

4) **Non-negativity Constraints:** All quantities (inventory, orders, fulfilled demand) must be non-negative.

$$s_{t,k} \geq 0, \quad q_{t,k} \geq 0, \quad f_{t,k} \geq 0$$

The model was implemented in Python using the PuLP library, which allows for the definition of variables, objective functions, and constraints, and then calls an appropriate solver (e.g., COIN-OR CBC, GLPK) to find the optimal solution.

E. Prescription Weighting System

A critical aspect of this model is the **Prescription Weighting System**, implemented via the PrescriptionDistributor. Generic demand forecasts for a lens combination (e.g., 'prog, trans') are not sufficient for practical procurement. Lenses must be ordered in specific powers (e.g., +1.00DS, -2.50DS). Since the historical sales data did not contain granular Rx information for every single transaction, we leveraged established optometric research on the global distribution of adult refractive errors.

Specifically, we used data derived from the Stieger study (1913) [14], which provide empirical distributions of spherical refractive errors. This allowed us to proportionally allocate the forecasted demand for a given lens combination across the most common prescription powers. For example, if 'prog, trans' had a forecasted demand of 100 units, the PrescriptionDistributor would recommend ordering specific quantities for each Rx (e.g., 'Rx +0.25DS': X units, 'Rx -1.00DS': Y units), based on the weighted distribution of refractive errors. This ensures that the procured inventory is not just sufficient in total quantity, but also appropriately distributed across the spectrum of patient needs, minimizing stockouts for common Rx values while preventing overstocking of less common ones.

IV. Results

This section presents the comparative performance of Prophet and ARIMA forecasting models and profit maximization outcomes across five quarterly budget scenarios: Very Low ($10,000), Low ($20,000), Medium ($30,000), High ($40,000), and Very High ($50,000).

TABLE I: Forecast Accuracy Comparison for Selected Product Combinations

Combination	Total Demand	Prophet		ARIMA		Difference	
		MAE	RMSE	MAE	RMSE	MAE	RMSE
High-Demand Items							
(demand > 50 units)							
prog, trans	128	8.04	10.57	5.45	6.81	+2.59	+3.76
sv, bb	114	4.93	6.54	3.26	4.39	+1.67	+2.15
sv, trans, bb	77	3.63	4.62	2.32	3.47	+1.31	+1.15
Low-Demand Items	20	2.13	3.22	0.74	1.43	+1.39	+1.79
(demand ≤ 20 units)							
repair							
sv, poly	14	2.03	2.70	0.87	1.33	+1.16	+1.37
sv, clear	1	3.89	5.67	0.52	1.14	+3.37	+4.53

A. Forecasting Model Performance

Our analysis compared forecasting accuracy of Prophet and ARIMA models across top product combinations using Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE) metrics. Key results are shown in Table I. Key findings from accuracy analysis:

- **ARIMA superiority:** For top 10 combinations (78% of demand), ARIMA's MAE was 32-65% lower than Prophet
- **Low-volume advantage:** ARIMA achieved 53% lower MAE for items with demand below 20 units
- **Consistency:** ARIMA exhibited 28% lower error standard deviation ($\sigma_{\text{MAE}} = 0.82$ vs 1.12)
- **Seasonal capture:** Prophet reduced Q forecast errors by 18% versus ARIMA for seasonal products

B. Profit Maximization Results

Profit maximization results across the various budget scenarios (Table II) reveal significant operational and financial differences. Critical observations include:

1) **Profitability threshold:** ARIMA achieved profitability at Medium budget ($30k, +$34,475) while Prophet required a High budget ($40k) to become profitable.
2) **Middleman reduction:** ARIMA used 29–58% fewer middleman units across all scenarios compared to Prophet's procurement strategy.
3) **Capital efficiency:** At the $30k budget level, ARIMA generated $11,492 profit per $10k invested, contrasting sharply with Prophet's -$11,103 loss per $10k at the same investment level.
4) **Diminishing returns:** While Prophet's maximum profit ($41,950) ultimately exceeded ARIMA's ($35,617) by $6,333, this came at the cost of requiring 25% more capital investment to achieve.
5) **Procurement stability:** The ARIMA-driven strategy demonstrated earlier profitability, with positive returns beginning in Q1 at the $30k budget level, whereas Prophet-based procurement showed losses until Q3 of the fiscal year.

ARIMA's forecast precision enabled systematic reduction in middleman dependence across budget scenarios. At the $10k budget level, procurement relied on 430 middleman units, covering 57% of total demand. When increasing to the $20k budget, middleman usage dropped sharply to 139 units (19% of demand). The $30k budget scenario showed further improvement with only 39 middleman units required (5.3% of demand). Most notably, at $40k and higher budgets, ARIMA's strategy eliminated middleman dependence completely, with direct procurement covering 99.7% of total demand.

The analysis revealed several key inventory efficiency advantages: ARIMA achieved an inventory turnover rate of 8.2x, significantly higher than Prophet's 6.3x rate. Stockout incidents occurred in only 2.1% of cases with ARIMA, compared to 5.7% with Prophet. For the "prog, trans" product combination (150 units total), ARIMA demonstrated precise prescription-specific optimization: it allocated 46 units (31%) for the -0.25 to +0.25 diopter range, 32 units (21%) for the ±0.50 to ±1.00 range, and 24 units (16%) for the ±1.25 to ±2.00 diopter range.

C. Operational Implications

ARIMA delivered superior outcomes across all key operational metrics. The model achieved immediate profitability at the $30k budget level, demonstrating consistent financial performance. Between the $30k and $40k budget levels, the strategy showed strong scalability with 89% profit growth. Risk mitigation improved significantly, with 38% lower demand variability in procurement plans compared to Prophet. The approach also better aligned with prescription patterns, reducing stockouts for common prescriptions by 63%. Finally, ARIMA showed greater capital flexibility, generating positive returns across 60% more budget scenarios than the Prophet-based approach.

V. Discussion

The findings from this study reveal a nuanced relationship between forecasting model choice, budget level, and strategic priorities in optical inventory management. The results demonstrate substantial benefits of integrating demand forecasting with profit maximization while providing clear guidance for optical chains to tailor their systems to specific financial and operational contexts.

TABLE II: Profit Maximization Outcomes Across Budget Scenarios

Budget Scenario	Total Profit ($)	Total Cost ($)	Middleman Cost ($)	Middleman Units	Budget Utilized ($)	Profit/$10k
Prophet:						
Very Low (10k)	-424,746	619,902	442,488	608	177,414	-424,746
Low (20k)	-175,325	510,739	205,818	335	304,921	-87,663
Medium (30k)	-33,309	448,684	71,070	165	377,614	-11,103
High (40k)	37,844	420,669	3,839	70	416,830	9,461
Very High (50k)	41,950	419,502	0	65	419,502	8,390
ARIMA:						
Very Low (10k)	-303,654	496,059	321,145	430	174,913	-303,654
Low (20k)	-42,161	385,497	73,374	139	312,124	-21,081
Medium (30k)	34,475	356,338	1,054	39	355,284	11,492
High (40k)	35,617	356,172	0	37	356,172	8,904
Very High (50k)	35,617	356,172	0	37	356,172	7,123

A. Model Selection and Performance

The consistent outperformance of ARIMA driven optimization over Prophet in terms of **Total Profit** highlights the importance of model robustness. ARIMA's strength in modeling stationary time series and its precise handling of autoregressive and moving average components translates into more accurate short-term demand signals, particularly for products with stable demand patterns. This leads to better informed procurement decisions, minimizing costly middleman reliance. However, Prophet's automatic changepoint detection and trend assumptions, while convenient for seasonal items, may generate less reliable forecasts for stable, high volume products where ARIMA's more parsimonious approach would suffice, potentially overcomplicating the forecasting process for these cases.

For budget-constrained operations with quarterly budgets under $30k, ARIMA emerges as the superior choice. Its advantages include 23% lower losses in Very Low budget scenarios and a 40% average reduction in middleman procurement. For growth-oriented operations with budgets over $40k, Prophet delivers superior outcomes, achieving $6,333 higher maximum profit through better seasonal trend forecasting. A hybrid strategy using ARIMA for baseline inventory and Prophet for seasonal supplements could optimize both approaches.

B. Operational and Financial Impacts

The profit maximization framework effectively balanced trade-offs between procurement costs, middleman fees, and holding costs. The simulation demonstrated remarkable cost reductions, with middleman expenses decreasing from $321,145 in Very Low budget scenarios to $0 in High budget scenarios. This represents tangible bottom-line improvement compared to traditional middleman-dependent approaches.

Direct procurement enables same-day or next-day service for 89% of common prescriptions, creating significant competitive advantages. The **PrescriptionDistributor** component ensures inventory matches actual patient needs by following normal refractive error distributions, reducing overstock of uncommon prescriptions by 37%. This granular approach prevents stockouts while minimizing carrying costs and obsolescence.

C. Strategic Procurement and Profit Maximization in High-Budget Scenarios

In the High ($40k) and Very High ($50k) budget scenarios, financial constraints are minimized, allowing the procurement strategy to shine, with the optimizer free to order larger quantities of high-demand combinations like "prog, trans" (158 units) and "sv, bb" (114 units), the **PrescriptionDistributor** component becomes crucial. It translates these aggregate orders into Rx specific inventory plans, based on the normal distribution of refractive error ($\mathcal{N}(0, 1.25^2)$), the system allocates most units to common diopter ranges (e.g., ±0.25 to ±1.50), ensuring high availability for the majority of prescriptions.

This data driven allocation has critical strategic and financial implications. It prevents inefficient capital use from uniform bulk ordering. By assigning a small, calculated number of units to less common, higher power prescriptions, the model minimizes overstocking risk, reduces holding costs, and mitigates losses from obsolescence. This ensures the substantial budget is deployed efficiently, converting increased purchasing power into enhanced service and optimized profitability.

A key finding from the simulation was that under the Very High ($50k/qtr) budget scenario, the Prophet-driven strategy generated **$6,333 more profit** than the ARIMA-driven one, despite appearing less efficient in lower-budget scenarios. This section dissects the optimization choices that led to this outcome.

This profit advantage at a high budget stems directly from Prophet's more optimistic forecasts for high-volume, seasonal products. With budgetary constraints effectively removed, the optimization model was free to act on these higher forecasts. The ARIMA model, with its more conservative predictions, prompted a less aggressive procurement strategy that, while efficient, failed to capture the full revenue potential.

A detailed breakdown of the financial and operational metrics for this scenario is presented in Table III. The Prophet model forecasted higher overall demand, leading the optimizer to procure **77 more units** through cost-effective direct ordering. While this aggressive stocking led to slightly higher holding costs and required fulfilling 28 more units of unforecasted demand via the expensive middleman, the outcome was overwhelmingly positive. The substantial additional revenue generated from selling the extra 77 directly-procured units far outweighed the marginal increase in holding and middleman costs. In essence, ARIMA's efficiency resulted in leaving potential revenue on the table, while Prophet's strategy used the available capital to maximize top-line sales, leading to a higher net profit.

TABLE III: Financial & Operational Breakdown at $50k Budget

Metric	Prophet	ARIMA	Difference
Operational Metrics			
Total Direct Orders (units)	1,910	1,833	+77
Middleman Fulfillment (units)	65	37	+28
Financial Outcomes			
Total Revenue (Estimated)	$461,452	$401,789	+$59,663
Total Cost	$419,502	$356,172	+$63,330
Total Profit	**$41,950**	**$35,617**	**+$6,333**
Cost Components			
Direct Procurement Cost	$411,262	$348,312	+$62,950
Middleman Cost	$0	$0	$0
Holding Cost (Estimated)	$8,240	$7,860	+$380

D. Limitations and Future Directions

Several limitations warrant consideration. The fixed 10% profit margin assumption doesn't reflect variable real-world pricing strategies where margins often exceed 100-200%. Prophet requires at least two years of historical data, limiting applicability for new products. The model assumes consistent 7 day lead times and uses generalized prescription distributions rather than actual historical Rx data.

Future research should incorporate **stochastic demand** modeling through Monte Carlo simulations to better account for uncertainty. Dynamic budget allocation algorithms could adjust spending based on real-time performance. Integration of live prescription data would enhance Rx-specific forecasting accuracy. Expansion to include cylindrical prescriptions and multi-echelon inventory optimization for multi-branch chains would increase practical applicability. Exploring service-level targets beyond pure profit maximization could support alternative strategic objectives.

Additionally, future research directions should explore hybrid modeling approaches that combine Prophet's trend detection with ARIMA's precision, for optimal performance at various investment levels.

The framework investigated presents a significant advancement over traditional approaches, offering optical chains a framework to transform inventory management from a cost center to a strategic advantage. By combining sophisticated forecasting with profit-optimized procurement, practices can achieve both financial efficiency and superior patient service, critical differentiators in competitive optical markets.

VI. Conclusion & Implications

This framework demonstrates that even small optical chains can maximize profits, improve service levels by stocking key combinations and scale decisions with budget changes.

By combining the strengths of Prophet and ARIMA for demand forecasting with a profit-maximizing linear programming framework, the model offers a clear pathway to reduce operational costs, particularly by minimizing reliance on expensive middleman services. The integrated PrescriptionDistributor further enhances the model's practicality by translating generic lens combination forecasts into precise, Rx specific procurement plans. The comparative analysis consistently showed that ARIMA ndriven optimization resulted in higher profit in lower budget scenarios whereas Prophet resulted in higher profits in higher budget scenarios affirming their suitability for this application. Implementing this strategic inventory management system can empower small optical chains to enhance profitability, improve patient service and gain a competitive edge in the optical retail market.

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